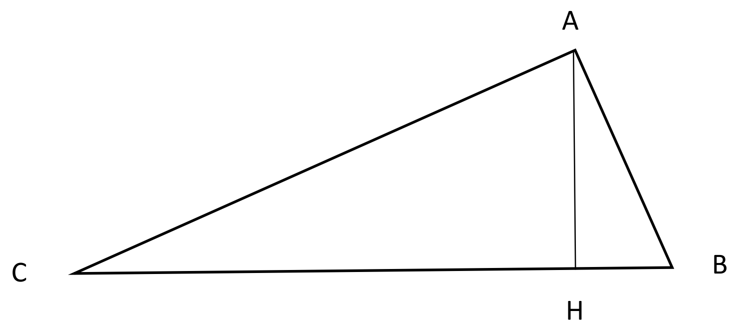
I'm going to prove that, in any quadrilateral ABCD, the sum of the lenghts of the three longest edges is greater than the sum of the length of the two diagonals.

I'll start by proving that in any triangle ABC in which angle BAC is right or obtuse and H is the intersection of the altitude from point A with the side BC, we have:

$AH + BC > AB + AC$

Let's first prove this for a right triangle.



By the Pythagorean theorem we have:

$AB^2 + AC^2 = BC^2$

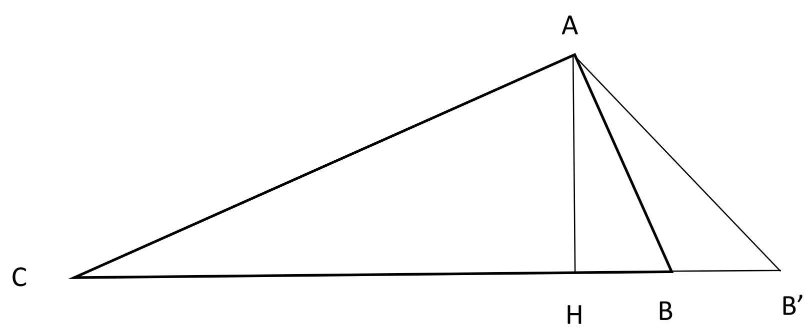
$AB^2 + AC^2 + 2*AB*AC = BC^2 + 2*AH*BC$

$AB^2 + AC^2 + 2*AB*AC < BC^2 + 2*AH*BC + AH^2$

$(AB + AC)^2 < (BC + AH)^2$

$AB + AC < BC + AH$

For an obtuse angle CAB' we draw the right angle CAB where B is between C and B':



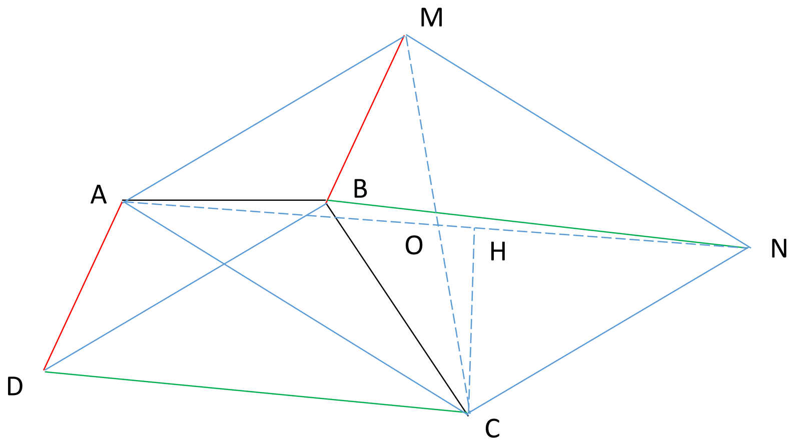
$AB + AC < BC + AH$

$AB' < AB + BB'$

Adding the 2 gives:

$AC + AB' < CB' + AH$

Let ABCD be the quadrilateral. We add the following segments:



- AM || BD and AM = BD. ADBM is a parallelogram so BM || AD and BM = AD
- CN || BD and CN = BD. CDBN is a parallelogram so BN || CD and BN = CD

Because AM || CN and AM = CN then ACNM is a parallelogram so MN || AC and MN = AC

We also draw diagonals AN and CM in the ACNM parallelogram and O is the intersection of the two.

Parallelogram ACNM has the side lengths equal to the length of the 2 diagonals. BA, BM, BN, BC have lengths equal to the 4 sides of the quadrilateral. Point B may be inside ACNM or outside it (when ABCD is not convex) but this does not affect the arguments below.

In order to prove that the 3 longest sides are longer than the 2 diagonals it is sufficient to prove that we can choose 3 out of the 4 segments (BA, BM, BN, BC) such that their length is greater than sum of the 2 distinct sides of ACNM.

We look at the pair of angles >= 90^o in ACNM (in our figure AMN and ACN). We choose the triangle which places B and the vertex of the obtuse angle on different sides of the longer side (ACN in our case). It is clear that BA + BN >= AN. Because B and C are on different sides of AN then CB >= CH.

The first part of the proof shows that AN + CH > AC + CN. So BA + BN + BC > AC + CN.

There are 3 segments which total a length greater that the 2 diagonals. So the longest 3 segments always total a length greater than the 2 diagonals.